Hessa Almheiri

HW 3 section of 13

Q1) $x$ is odd, $y$ is even.

a) show $x+y$ is odd (direct) let $x$ be odd integer & $y$ even integer

hence $x = (2n+1)$ and $y = (2m)$ for some $n, m \in \mathbb{Z}$

$(2n+1) + 2m = 2n+2m+1 = 2\underbrace{(n+m)}_{c}+1 \quad c \in \mathbb{Z}$

$= 2c+1$ is odd, thus $x+y$ is an odd integer

b) show $xy$ is even (direct)

let $x$ be an odd integer and $y$ be an even integer.

Hence $x = (2n+1)$, and $y = (2m)$ for some $n, m \in \mathbb{Z}$.

$= (2n+1)(2m) = 4nm + 2m = 2\underbrace{(2nm+m)}_{c} \quad c \in \mathbb{Z}$

$= 2c$, thus $xy$ is an even integer

c) show $y^2$ is even (direct)

let $y$ be an even integer Hence $y = 2n \quad n \in \mathbb{Z}$

$y^2 = (2n)^2 = 4n^2 = 2\underbrace{(2n^2)}_{m} \quad m \in \mathbb{Z}$

$= 2m$, thus $y^2$ is even

d) show $x^2$ is odd (direct)

let $x$ be an odd integer Hence $y = 2n+1 \quad n \in \mathbb{Z}$

$x^2 = (2n+1)^2$

$x^2 = 4n^2+4n+1$

$= 2\underbrace{(2n^2+2n)}_{m}+1 \quad m \in \mathbb{Z}$

$= 2m+1$, thus $x^2$ is odd.

Q) $x$ is irrational, $y$ is rational

show $x \pm y$ is irrational (contradiction)

Deny $x+y = z \in \mathbb{Q}$ hence its rational

$\underbrace{x}_{\text{irrational}} = \underbrace{z-y}_{\text{rational}}$, contradiction

Hence denial is invalid, thus $x+y$ is irrational

Deny $x-y = z \in \mathbb{Q}$ hence its rational

$\underbrace{x}_{\text{irrational}} = \underbrace{z+y}_{\text{rational}}$, contradiction.

Hence denial is invalid, thus $x-y$ is irrational.

Q2) let $x$ be an odd integer

a) show $x = 2n-1$ for some integer (direct)

let $x$ be an odd integer,

$x = 2m+1$ for some $m \in \mathbb{Z}$

$x = 2(m+1-1)+1$

$x = 2(m+1)-2+1$

$x = 2\underbrace{(m+1)}_{n} - 1$ for some $n \in \mathbb{Z}$

$x = 2n-1$. thus $x = 2n-1$ is odd.

Q3) show $\sqrt{3}+\sqrt{13}$ is irrational (contradiction)

1) Deny $\sqrt{3}+\sqrt{13}$ is rational

Thus $\sqrt{3}+\sqrt{13} = \frac{a}{b}$ $\gcd(a,b) = 1$ let $\frac{a}{b} = x$

$(\sqrt{3}+\sqrt{13})^2 = x^2$

$16 + 2\sqrt{39} = x^2$

$\frac{2\sqrt{39}}{2} = \frac{x^2-16}{2}$

$\underbrace{\sqrt{39}}_{\downarrow} = \frac{x^2-16}{2} \in \mathbb{Q}$ $\frac{x^2-16}{2}$ is rational

prove using 4 method

2) Deny $\sqrt{39}$ is rational

Hence $\sqrt{39} = \frac{a}{b}$ $\gcd(a,b) = 1$

$39 = \frac{a^2}{b^2}$, thus $a^2, b^2$ are odd hence $a, b$ are odd integers

$a = 2n+1$ $b = 2m+1$ for some $n, m \in \mathbb{Z}$

$39 = \frac{(2n+1)^2}{(2m+1)^2}$ $39 = \frac{(4n^2+4n+1)}{4m^2+4m+1}$

$39(4m^2) + 39(4m) + 39 = 4n^2+4n+1$

$\frac{39(4m^2) + 39(4m) + 39 - 1}{4} = \frac{4n^2+4n}{4}$

$\underbrace{39m^2 + 39m + \frac{38}{4}}_{\text{not integer}} = \underbrace{n^2+n}_{\text{integer}}$, contradiction

Hence denial is invalid, thus $\sqrt{39}$ is irrational

$\underbrace{\sqrt{39}}_{\text{irrational}} = \underbrace{\frac{x^2-16}{2}}_{\text{rational}}$, invalid statement

Rami Mouradi

Question: $x$ is odd, $y$ is even. show $x+y$ is odd (by direct) proof.

let $x$ be odd and $y$ be even

thus $x = 2n+1$ and $y = 2m$ for some integers $n, m \in \mathbb{Z}$

$$x+y = 2n+1 + 2m$$
$$= 2\underbrace{(n+m)}_{d \text{ (integer)}} + 1$$
$$= 2d+1, \quad d \in \mathbb{Z}$$

thus $x+y$ is an odd integer

Question: show $x^2$ is even. (direct)

let $x$ be even

thus $x = 2m$ for some $m \in \mathbb{Z}$

$$x^2 = (2m)^2$$
$$= 4m^2$$
$$= 2\underbrace{(2m^2)}_{\text{(integer) } d}$$
$$= 2d$$

thus $x^2$ is even

Question: Show $y^2$ is odd. (direct)

Let $y$ be odd

Thus $y = 2m+1$ for some integer

$m \in \mathbb{Z}$

$$y^2 = (2m+1)^2$$

$$= 4m^2 + 4m + 1$$

$$= 4\underbrace{(m^2+m)}_{n \text{ (integer)}} + 1$$

$4n$

$$= 2\underbrace{(2n)}_{L \text{ (integer)}} + 1$$

$$= 2L + 1$$

Thus $y^2$ is odd

Question: $\sqrt{3} + \sqrt{13}$ is irrational (contradiction)

Solution: deny

Thus $\sqrt{3} + \sqrt{13}$ is rational

$$\sqrt{3} + \sqrt{13} = x \in \mathbb{Q}.$$

square it

$$16 + 2\sqrt{39} = x^2 \in \mathbb{Q}.$$

$$2\sqrt{39} = x^2 - 16.$$

$$\sqrt{39} = \frac{x^2-16}{2} \in \mathbb{Q}$$

Use 4-method to show $\sqrt{39}$ is irrational.

Thus $\sqrt{39}$ is rational

hence $\sqrt{39} = \frac{a}{b}$, $\gcd(a,b) = 1$.

$39 = \frac{a^2}{b^2}$, $a^2$ and $b^2$ are odd

hence $a$ and $b$ are odd.

$a = 2m+1$, $b = 2n+1$ for some $m, n \in \mathbb{Z}$

$$39 = \frac{(2m+1)^2}{(2n+1)^2}$$

$$39 = \frac{4m^2+4m+1}{4n^2+4n+1}$$

$39(4n^2) + 39(4n) + 39 = 4m^2 + 4m + 1$ divide by 4

$$\underbrace{39n^2 + 39n + \frac{38}{4}}_{\text{is not an integer}} = \underbrace{m^2+m}_{\text{integer}}$$

hence our denial is invalid.

Thus $\sqrt{39}$ is irrational as well

as $\sqrt{3}+\sqrt{13}$ is irrational.

Question: let $x$ be an odd integer, show $x = 2n-1$ for some integer $n$ (direct)

let $x$ be odd.

thus $x = 2m+1$ for some $m \in \mathbb{Z}$

$x = 2m+1$ add 0 to anything rule

$x = 2(m+1-1)+1$

$x = 2\underline{(m+1)} - 2 + 1$

$x = 2h - 1$, $h \in \mathbb{Z}$

thus $x$ is odd

Remark: irrational + irrational could be rational or irrational

irrational (any operation) irrational depends on the numbers given.

Question: $x$ is irrational, $y$ is rational

Show that $x \pm y$ is irrational (contradiction)

Solution: deny

hence $x + y$ is rational

thus $x + \frac{a}{b} = \frac{m}{n}$

$x = \frac{m}{n} - \frac{a}{b}$ common denominator

$$X = \frac{\boxed{mb - an}}{\boxed{nb}}$$ integer

integer

not integer

but $x$ is not rational

hence our denial is invalid.

thus $x + y$ is irrational

Example: $-\sqrt{2} + (2 + \sqrt{2}) = 2$

rational